Sydney Girls High School
2016
Trial Higher School Certificate
Examination

Mathematics Extension 1

General Instructions

- Reading Time – 5 minutes
- Working time – 2 hours
- Write using black or blue pen
  Black pen is preferred
- Board-approved calculators may be used
- In Questions 11 – 14, show relevant mathematical reasoning and/or calculations.
- A mathematics exam reference sheet is provided.

This is a trial paper ONLY. It does not necessarily reflect the format or the contents of the 2016 HSC Examination Paper in this subject.

Total marks – 70

SECTION 1 –

10 marks

- Attempt questions 1 – 10
- Answer on the Multiple Choice sheet provided
- Allow about 15 minutes for this section

SECTION II –

60 marks

- Attempt questions 11 – 14
- Answer on the blank paper provided
- Begin a new page for each question
- Allow about 1 hours 45 minutes for this section

Name: ________________________________

Teacher: ______________________________

Section I

10 Marks

Attempt Questions 1 – 10

Allow about 15 minutes for this section.

Use the multiple choice answer sheet for Questions 1 -10

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1. What is the remainder when $x^3 + 8x$ is divided by $x - 4$

(A) $x^2 + 4x + 24$

(B) $x^2 - 2x$

(C) 96

(D) -96

2. Given that $\dfrac{dN}{dt} = k(N - 50)$, which expression is equal to N ?

(A) $-50 - 70e^{kt}$

(B) $50 + 70e^{kt}$

(C) $-70 - 50e^{kt}$

(D) $70 + 50e^{kt}$

3. Two secants from the Point P intersect a circle as shown in the diagram.

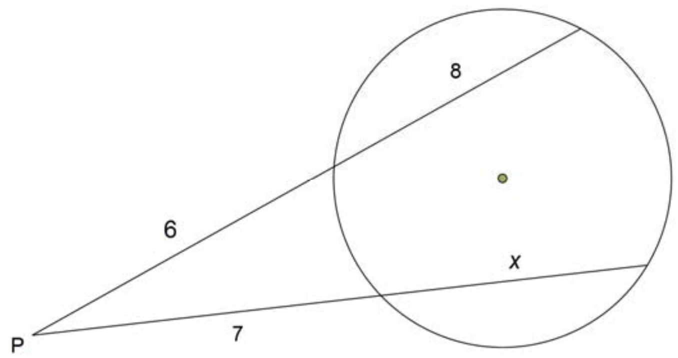


Figure is not to scale.

What is the value of x ?

(A) 5

(B) $\dfrac{41}{7}$

(C) 7

(D) $\dfrac{28}{3}$

4. A team consists of 5 women and 3 men. The women are chosen from a group of 9 while the men are chosen from a group of 6. In how many ways could the team be selected?

(A) ${}^9P_5 \times {}^6P_3$

(B) ${}^9C_5 \times {}^6C_3$

(C) ${}^9P_5 + {}^6P_3$

(D) ${}^9C_5 + {}^6C_3$

5. What are the asymptotes of $y = \dfrac{x-4}{(x-2)(x+3)}$?

(A) $y = 4, x = 3, x = -3$

(B) $y = 0, x = -2, x = 3$

(C) $y = 4, x = -2, x = 3$

(D) $y = 0, x = 2, x = -3$

6. What is the range of the function $f(x) = 2\cos^{-1} x$?

(A) $0 \le y \le 2\pi$

(B) $-2 \le y \le 2$

(C) $0 \le y \le \pi$

(D) $-\frac{1}{2} \le y \le \frac{1}{2}$

7. What is the value of k such that $\int_0^k \frac{1}{x^2+9}\, dx = \frac{\pi}{9}$?

(A) $\sqrt{3}+3$

(B) $\frac{\sqrt{3}}{3}$

(C) $\sqrt{3}$

(D) $3\sqrt{3}$

8. What is the value of $\lim_{x \to 2} \frac{\sin(2-x)}{x-2}$?

(A) 0

(B) 1

(C) -1

(D) Undefined

9. The acceleration of a particle at a position x on the number line is given by $\ddot{x} = -25x$. Initially the particle is at the origin and has a velocity of $10\,\text{ms}^{-1}$.

Find the velocity after $\dfrac{\pi}{15}$ secs.

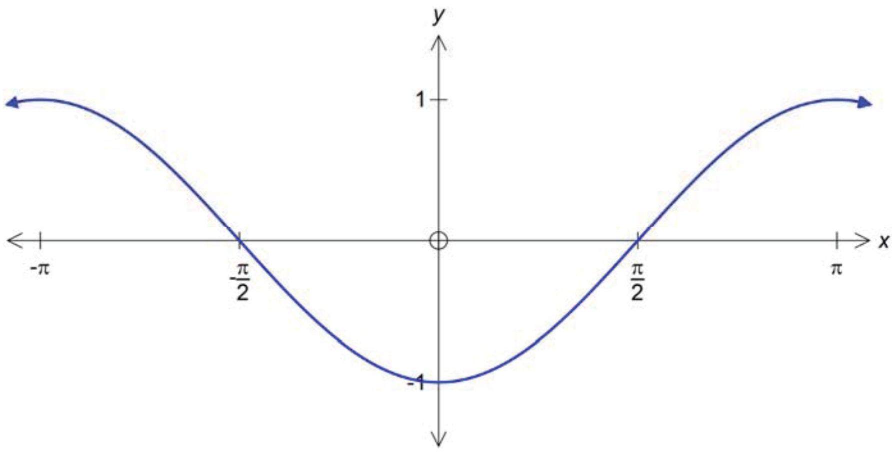
(A) $5ms^{-1}$

(B) $-5ms^{-1}$

(C) $\sqrt{3}ms^{-1}$

(D) $\dfrac{-5\pi}{3}ms^{-1}$

10.



The equation of the above graph is :

(A) $y = \sin\left(x + \dfrac{\pi}{2}\right)$

(B) $y = \sin\left(x - \dfrac{\pi}{2}\right)$

(C) $y = \sin\left(\dfrac{\pi}{2} - x\right)$

(D) $y = \cos(-x)$

Section II

60 Marks
Attempt Questions 11-14
Allow about 1 hour 45 minutes for this section
Start each question on a new page
In Questions 11 -14, your responses should include relevant mathematical reasoning and/or calculations

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Question 11 (15 marks) Start a new page

(a) Find $\int e^{\log x} dx$. 2

(b) Find the coordinates of the point P which divides the interval joining (-1,3) and (4,-2) in the ratio 2:3. 2

(c) Solve the inequality $\frac{5}{1-x} \le 2$. 3

(d) Differentiate $\cos^3 x$. 2

(e) Use the substitution $u = 3x^2 - 2$ to evaluate $\int_{\sqrt{2}}^{\sqrt{6}} x\sqrt{3x^2-2} \quad dx$. 3

(f) The zeros of the polynomial $P(x) = 2x^3 - 6x^2 + 7x + 1$ are α, β and γ. Find:

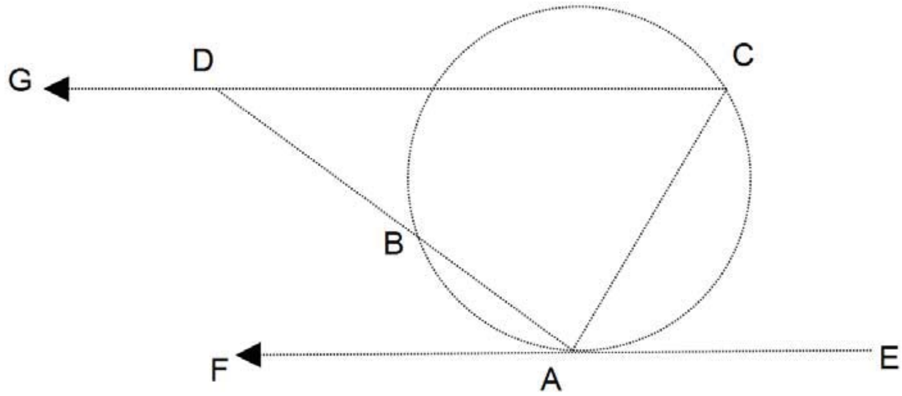
(i) $\alpha + \beta + \gamma$ 1

(ii) $\alpha\beta + \alpha\gamma + \beta\gamma$ 1

(iii) $\alpha^2 + \beta^2 + \gamma^2$ 1

Question 12 (15marks) Start a new page

(a) In the diagram A, B and C are points on the circle. The line AE is a tangent to the circle at A, and AB is produced to D so that DC is parallel to AE.
Copy the diagram.



(i) Show that $A\hat{C}B = A\hat{D}C$, giving reasons. 2

(ii) Deduce that $AC^2 = AB \times AD$. 2

(b) The points $P(2ap, ap^2)$ and $Q(2aq, aq^2)$ lie on the parabola $x^2 = 4ay$.

(i) Show that the equation of chord PQ is $y - \frac{1}{2}(p+q)x + apq = 0$. 1

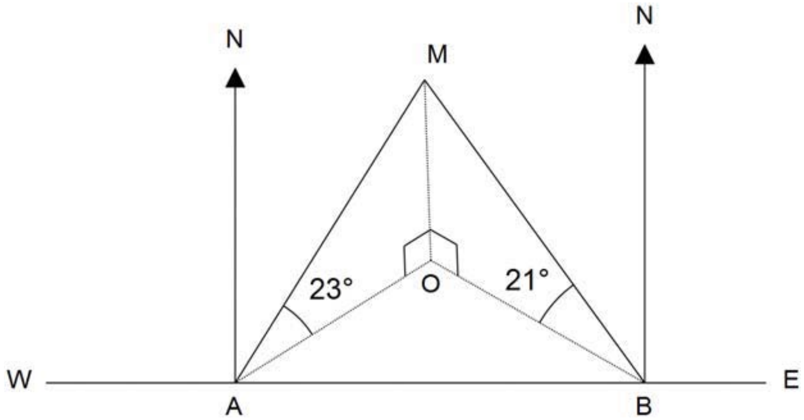
(ii) Show that if PQ is a focal chord then $pq = -1$. 1

(iii) If PQ is a focal chord and P has coordinates $(4a, 4a)$ what are the coordinates of the midpoint of PQ in terms of a ? 2

Question 12 (continued)

(c) A person walks 3000m due east along a road from point A to point B. A mountain OM, where M is the top of the mountain is on a bearing of 053° from A and 319° from B. The point O is directly below point M and is on the same horizontal plane as the road. The height of the mountain above point O is h metres.

From point A, the angle of elevation to the top of the mountain is 23°.
From point B, the angle of elevation to the top of the mountain is 21°.

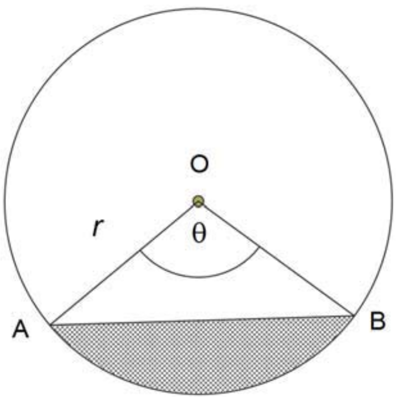


(i) Show that $OA = \dfrac{h}{\tan 23^\circ}$. 1

(ii) Find $A\hat{O}B$. 1

(iii) Hence, find the value of h.(correct to 2 decimal places) 2

(d) In the circle below with centre O, the shaded region has the same area as the triangle AOB.



(i) Show that $\theta = 2\sin\theta$. 1

(ii) Taking $\theta_1 = \dfrac{\pi}{2}$ as a first approximation to the value of θ, use one application of Newton's method to find a second approximation to the value of θ. 2

Question 13 (15 marks) Start a NEW page

(a) A particle moves in a straight line and its position at time t seconds is given by $x = 3 + \sin 4t + \cos 4t$.

(i) Express $\sin 4t + \cos 4t$ in the form $R\sin(4t+\alpha)$ where α is in radians. 2

(ii) The particle is undergoing simple harmonic motion. Find the amplitude and the centre of motion. 2

(iii) When does the particle first reach its minimum speed after $t = 0$? 1

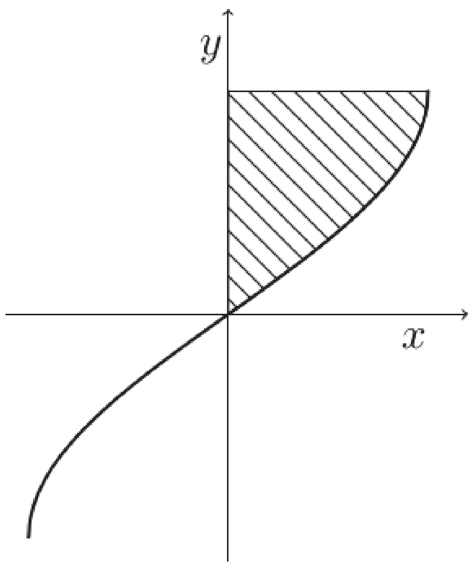
(b) Evaluate $\int_0^{\frac{\pi}{4}} \sin x \cos^2 x \ dx$. 2

(c) Use mathematical induction to prove that $1 + 3 + 6 + \dots\dots + \frac{1}{2}n(n+1) = \frac{1}{6}n(n+1)(n+2)$

for all integers $n = 1, 2, 3, \dots$ 3

(d) The sketch below shows the graph of the curve $y = f(x)$ where $f(x) = 3\sin^{-1}\frac{x}{2}$.

The area between the curve and the $y-$axis in the first quadrant is shaded.



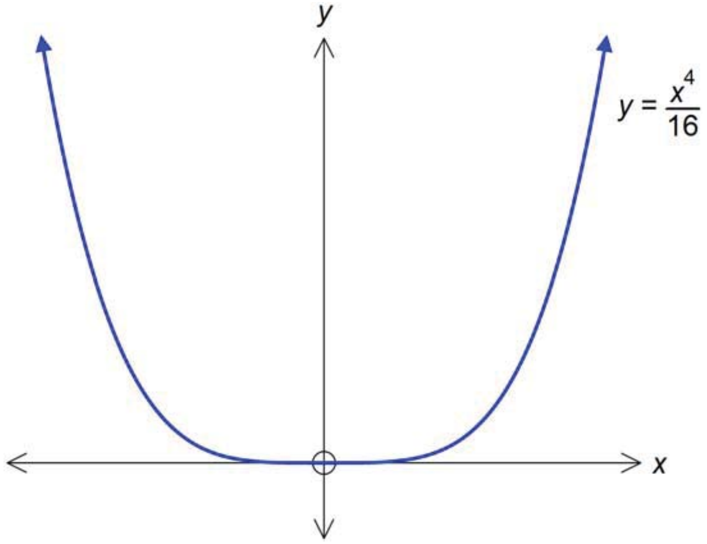
(i) Find the maximum value of $f(x)$. 1

(ii) Determine the inverse function $y = f^{-1}(x)$ and write down the domain of this inverse function. 2

(iii) Calculate the area of the shaded region. 2

Question 14 (15 marks) Start a NEW page

(a) A water tank is generated by rotating the curve $y = \frac{x^4}{16}$ around the $y-$axis.



(i) Show that the volume of water, V as a function of its depth h, is given by $V = \frac{8}{3}\pi h^{\frac{3}{2}}$. 2

(ii) Water drains from the tank through a small hole at the bottom.
The rate of change of the volume of water in the tank is proportional to the square root of the water's depth.
Use this fact to show that the water level in the tank falls at a constant rate. 3

(b) A standard pack of 52 cards consists of 13 cards of the four suites: spades, hearts clubs and diamonds.

(i) In how many ways can five cards be selected without replacement so that exactly two are hearts and three are diamonds? (Assume that the order of selection of the five cards is not important.) 2

(ii) In how many ways can five cards be selected without replacement if at least four must be of the same suite? (Assume that the order of selection is not important) 2

Question 14 (continued)

(c) When a projectile is fired with velocity $V\ ms^{-1}$ at an angle θ above the horizontal, the horizontal and vertical displacements (in metres) from the point of projection at time t seconds are given by $x = Vt\cos\theta$ and $y = Vt\sin\theta - \frac{1}{2}gt^2$ respectively (where g is the acceleration due to gravity). You do NOT need to prove these two results.

The particle reaches a maximum height of H metres after T seconds.

(i) Find T in terms of V, θ and g. 1

(ii) Hence, show that when $t = \frac{1}{2}T$ the height of the particle is $\frac{3}{4}H$. 3

(iii) Show that when $t = \frac{1}{2}T$ the particle is moving on a path inclined at an angle α to the horizontal such that $\tan\alpha = \frac{1}{2}\tan\theta$. 2

End of paper

Sydney Girls High School

Mathematics Faculty

Multiple Choice Answer Sheet
2016 Trial HSC Mathematics Extension I

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample $2 + 4 = ?$ (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ~~●~~ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate this by writing the word *correct* and drawing an arrow as follows:

correct

A ~~●~~ B ~~●~~ C ○ D ○

Student Number: ANSWERS

Completely fill the response oval representing the most correct answer.

1. A ○ B ○ C ● D ○
2. A ○ B ● C ○ D ○
3. A ● B ○ C ○ D ○
4. A ○ B ● C ○ D ○
5. A ○ B ○ C ○ D ●
6. A ● B ○ C ○ D ○
7. A ○ B ○ C ○ D ●
8. A ○ B ○ C ● D ○
9. A ● B ○ C ○ D ○
10. A ○ B ● C ○ D ○

Ext 1 2016

Q11

a) $\int e^{\log x}\,dx$

$= \int x\,dx$

$= \frac{x^2}{2} + c$

* Many students did this question incorrectly. There is no linear function on e, so you can't divide by derivative of $\log x$.

b) $x = \frac{2(4)+3(-1)}{5}$

$y = \frac{2(-2)+3(3)}{5}$

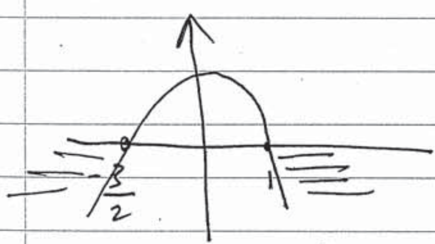
$x = 1$, $y = 1$

$P(1,1)$

c) $\frac{5}{1-x} - 2 \le 0$

$\frac{5-2(1-x)}{1-x} \le 0$

$(3+2x)(1-x) \le 0$



$x > 1$, $x \le -\frac{3}{2}$

Some students put $x \geq 1$ but $x \neq 1$.

d) $y = (\cos x)^3$

$y' = -3\cos^2 x \cdot \sin x$

e) $\frac{du}{dx} = 6x$

$du = 6x\,dx$

$x = \sqrt{2} \longrightarrow u = 6-2$

$u = 4$

$x = \sqrt{6} \longrightarrow u = 16$

$\frac{1}{6}\int_{\sqrt{2}}^{\sqrt{6}} 6x\sqrt{3x^2-2}\,dx$

$\frac{1}{6}\int_4^{16} u^{\frac{1}{2}}\,du$

$\frac{1}{6}\left[\frac{2u^{\frac{3}{2}}}{3}\right]_4^{16} = \frac{1}{6}\left[\frac{128-16}{3}\right]$

$= \frac{56}{9}$

f) i) $\alpha + B + \gamma = \frac{6}{2}$

$= 3$

ii) $\alpha B + \alpha\gamma + B\gamma = \frac{7}{2}$

iii) $\alpha^2 + B^2 + \gamma^2 = (\alpha + B + \gamma)^2 - 2(\alpha B + \alpha\gamma + B\gamma)$

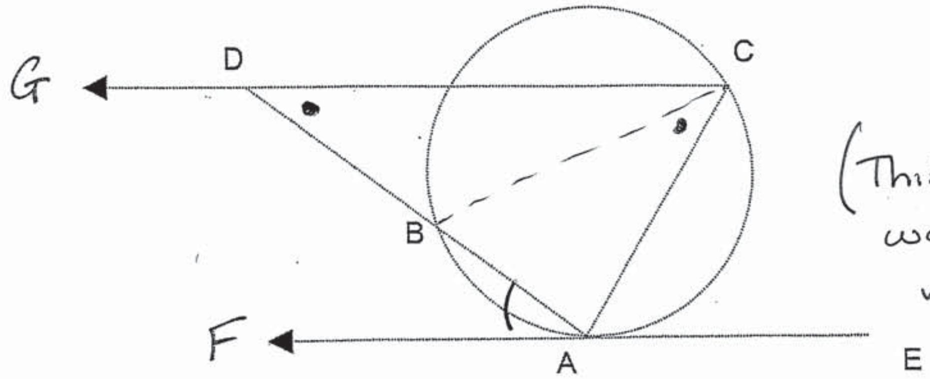
$= 3^2 - 2\left(\frac{7}{2}\right)$

$= 9 - 7$

$= 2$

Question 12 - Ext 1 - HSC TRIAL EXAM - 2016
(15 marks)

a) i)



(This was completed very well)

i) $\angle FAD = \angle ACB$ (angle in alternate segment)

$\angle FAD = \angle ADC$ (alternate $\angle$'s $DC \parallel FE$)

$\therefore \angle ACB = \angle ADC$

2

ii) In ΔADC and ΔACD

$A\hat{C}B = \angle A\hat{D}C$ (proven above)

$B\hat{A}C$ (common)

$\therefore \Delta ABC \parallel\!| \Delta ACD$

$$\frac{AC}{DA} = \frac{AB}{AC}$$

$\therefore AC^2 = AB \times AD$

2

(Many students didn't recognise to prove using similar Δ's)

(Section was poorly completed)

b) i) $P(2ap, ap^2)$ and $Q(2aq, aq^2)$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{aq^2 - ap^2}{2aq - 2ap}$$

$$= \frac{\cancel{a}(\cancel{q-p})(q+p)}{2\cancel{a}(\cancel{q-p})}$$

$$m = \frac{p+q}{2}$$

Equation of chord

$$y - y_1 = m(x - x_1)$$

$$y - ap^2 = \frac{p+q}{2}(x - 2ap)$$

$$y - ap^2 = \frac{1}{2}(p+q)x - ap(p+q)$$

$$y - \cancel{ap^2} = \frac{1}{2}(p+q)x - \cancel{ap^2} - apq$$

$$y - \frac{1}{2}(p+q)x + apq = 0 \quad 1$$

(This section was completed very well)

ii) If focal chord passes through $F(0, a)$

$$a - \frac{1}{2}(p+q)0 + apq = 0$$

$$a = -apq$$

$$\therefore pq = -1 \quad 1$$

(This section was completed very well).

iii) $P(4a, 4a)$ $Q(2aq, aq^2)$

$P(2ap, ap^2)$

$\therefore 2ap = 4a$ $\therefore pq = -1$

$p = 2$ $2q = -1$

$q = -\frac{1}{2}$

$$\therefore \text{Midpoint}\left(\frac{4a + 2aq}{2}, \frac{4a + aq^2}{2}\right)$$

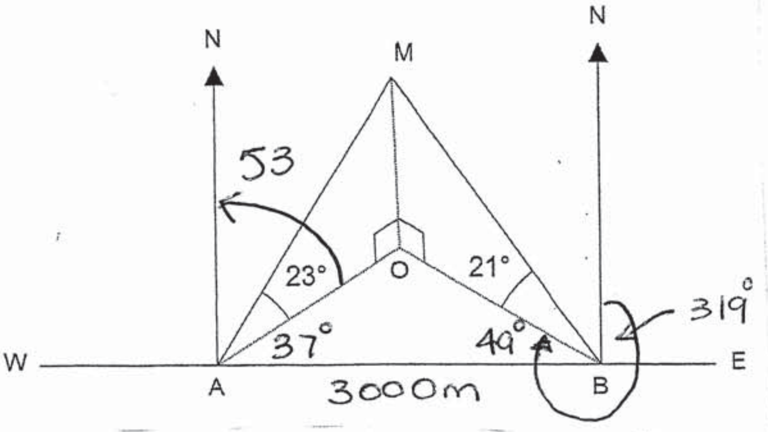
$$= \left(\frac{4a - a}{2}, \frac{4a + \frac{1}{4}a}{2}\right)$$

$$= \left(\frac{3a}{2}, \frac{17a}{8}\right)$$

2

(This section was completed very poorly. Most knew the midpoint formula but didn't know how to obtain a value for p & q and then find in terms of 'a'.)

c)



(This section was complete very well)

i) In ΔAOM, $\tan\theta = \frac{opp}{adj}$

$\tan 23^\circ = \frac{OM}{OA}$

$\tan 23^\circ = \frac{h}{OA}$

$\therefore OA = \frac{h}{\tan 23^\circ}$

1

ii) $\angle OAB = 90 - 53^\circ$, $\angle OBA = 319^\circ - 270^\circ$

$= 37^\circ$ $= 49^\circ$

$\therefore \angle AOB = 180 - (37 + 49)$

$= 94^\circ$.

1

(This was completed very poorly with most students not obtaining $A\hat{O}B$)

iii) $OB = \frac{h}{\tan 21^\circ}$

(Consequently they also didn't recognise that the cosine rule needed to be used. Very poorly completed as a section).

In ΔAOB,

$$3000^2 = OA^2 + OB^2 - 2(OA)(OB)\cos 94^\circ$$

$$3000^2 = \left(\frac{h}{\tan 23^\circ}\right)^2 + \left(\frac{h}{\tan 21^\circ}\right)^2 - 2\left(\frac{h}{\tan 23^\circ}\right)\left(\frac{h}{\tan 21^\circ}\right)\cos 94^\circ$$

$$3000^2 = h^2\cot^2 23^\circ + h^2\cot^2 21^\circ - 2h^2\cot 23^\circ \cot 21^\circ \cos 94^\circ$$

$$3000^2 = h^2(\cot^2 23^\circ + \cot^2 21^\circ - 2\cot 23^\circ \cot 21^\circ \cos 94^\circ)$$

$$\therefore h^2 = \frac{3000^2}{\cot^2 23^\circ + \cot^2 21^\circ - 2\cot 23^\circ \cot 21^\circ \cos 94^\circ}$$

$$h^2 = 682192.923$$

$$\therefore h \doteqdot 825.95\text{ m}.$$

2

d) i) Area of minor segment = Area of Δ

$$\frac{1}{2}\cancel{r^2}(\theta - \sin\theta) = \frac{1}{2}\cancel{r^2}\sin\theta$$

(This was completed well)

$$\theta - \sin\theta = \sin\theta$$

$$\theta = 2\sin\theta \qquad 1$$

ii) $$a_2 = a_1 - \frac{f(a_1)}{f'(a_1)}$$

$$f(\theta) = 2\sin\theta - \theta \qquad f'(\theta) = 2\cos\theta - 1$$

$$f\left(\frac{\pi}{2}\right) = 2\sin\frac{\pi}{2} - \frac{\pi}{2} \qquad f'\left(\frac{\pi}{2}\right) = -2\cos\frac{\pi}{2} - 1$$

$$= 2 - \frac{\pi}{2} \qquad = 0 - 1$$

$$= -1$$

$$\therefore a_2 = \frac{\pi}{2} - \left(\frac{2 - \frac{\pi}{2}}{-1}\right)$$

$$= \frac{\pi}{2} + 2 - \frac{\pi}{2}$$

$$a_2 = 2 \qquad 2$$

- This section was completed very poorly. Both $f(\theta)$ and $f'(\theta)$ was found incorrectly in most cases. Consequently substitution was incorrect. 1 mark was given if students realised $\frac{2}{0}$ was undefined and stated it in solution.

Q13

a) i) $\sin 4t + \cos 4t = R\sin(4t+\alpha)$

$R = \sqrt{1^2+1^2} = \sqrt{2}$

$\tan\alpha = 1 \quad \therefore \alpha = \frac{\pi}{4}$

$\sin 4t + \cos 4t = \sqrt{2}\sin\left(4t+\frac{\pi}{4}\right)$

ii) $x = 3 + \sqrt{2}\sin\left(4t+\frac{\pi}{4}\right)$ ①

$\dot{x} = 4\sqrt{2}\cos\left(4t+\frac{\pi}{4}\right)$

$\ddot{x} = -16\sqrt{2}\sin\left(4t+\frac{\pi}{4}\right)$

$\ddot{x} = -16(x-3)$ from ①

$\ddot{x} = -4^2(x-3)$

Thus the centre is $x=3$

$\dot{x}=0$ at the extremity

$4\sqrt{2}\cos\left(4t+\frac{\pi}{4}\right) = 0$

$\cos\left(4t+\frac{\pi}{4}\right) = 0$

$4t+\frac{\pi}{4} = \frac{\pi}{2}$

$t = \frac{\pi}{16} \rightarrow x = 3 + \sqrt{2}\sin\frac{\pi}{2}$

$x = 3+\sqrt{2}$ at the extremity

[A number of students identified the centre of motion is at $x=0$ without referring to the given Equation]

Amplitude $= 3+\sqrt{2}-3$

$= \sqrt{2}$

Q13

a/(iii) Min speed $\dot{x} = 0$

$$\dot{x} = 0$$

$$4\sqrt{2}\cos\left(4t + \frac{\pi}{2}\right) = 0$$

$$\cos\left(4t + \frac{\pi}{4}\right) = 0$$

$$4t + \frac{\pi}{4} = \frac{\pi}{2}$$

$$t = \frac{\pi}{16} \text{ sec}$$

b) $\displaystyle\int_0^{\pi/4} \sin x \cos^2 x \, dx$

Let $u = \cos x$ $\qquad x = \frac{\pi}{4} \longrightarrow u = \frac{1}{\sqrt{2}}$

$\frac{du}{dx} = -\sin x$ $\qquad x = 0 \longrightarrow u = 1$

$$I = -\int_1^{\pi/4} -\sin x \cdot \cos^2 x \, dx$$

$$= -\int_1^{1/\sqrt{2}} u^2 \, du$$

$$= -\left[\frac{u^3}{3}\right]_1^{1/\sqrt{2}} = \frac{4-\sqrt{2}}{12}$$

[some students got mixed up between differentiation and integration of trig functions. Use integration by subs is the easier method in this case.]

c) $1 + 3 + 6 + \cdots \frac{1}{2}n(n+1) = \frac{1}{6}n(n+1)(n+2)$

prove it is true for $n = 1$

$$\frac{1}{2}(1)(2) = \frac{1}{6}(1)(2)(3)$$

$$1 = 1$$

Assume it is true for $n = k$

$$1 + 3 + 6 + \cdots \frac{1}{2}k(k+1) = \frac{1}{6}k(k+1)(k+2)$$

prove it is true for $n = k+1$

$$1 + 3 + 6 + \cdots \frac{1}{2}k(k+1) + \frac{1}{2}(k+1)(k+2) = \frac{1}{6}(k+1)(k+2)(k+3)$$

$$LHS = \frac{1}{2}(k+1)(k+2) + \frac{1}{6}k(k+1)(k+2)$$

$$= \frac{3(k+1)(k+2) + k(k+1)(k+2)}{6}$$

$$= \frac{(k+1)(k+2)(k+3)}{6}$$

$$= RHS$$

It is true for $n = k+1$.

It is proven by Mathematical Induction, for $n \geqslant 1$

[Most students did well in this Induction question.]

d) $\boxed{Q13}$ $-\frac{\pi}{2} \times 3 \leq y \leq \frac{\pi}{2} \times 3$

i) $-\frac{3\pi}{2} \leq y \leq \frac{3\pi}{2}$

Max value of $f(x) = \frac{3\pi}{2}$

ii) $y = 3\sin^{-1}\frac{x}{2}$

$x = 3\sin^{-1}\frac{y}{2}$

$\frac{y}{2} = \sin\frac{x}{3}$

$y = 2\sin\frac{x}{3}$

D: $-\frac{3\pi}{2} \leq x \leq \frac{3\pi}{2}$

R: $-2 \leq y \leq 2$

iii) Area $= 2\int_0^{3\pi/2} \sin\frac{x}{3}\,dx$

$= 2\left[-3\cos\frac{x}{3}\right]_0^{3\pi/2}$

$= -6\left[\cos\frac{\pi}{2} - \cos 0\right]$

$= 6$ units^2

[A number of students used the incorrect limits or were unable to calculate the correct shaded Area.]

2016 THSC Ext 1

Q14

(a) $y = \frac{x^4}{16}$

(i) $x^2 = 4y^{\frac{1}{2}}$

$V = \pi \int_0^h x^2 \, dy$

$V = 4\pi \int_0^h y^{\frac{1}{2}} \, dy$

$V = 4\pi \left[\frac{2y^{\frac{3}{2}}}{3} \right]_0^h$

$V = \frac{8\pi h^{\frac{3}{2}}}{3}$

(ii) $\frac{dV}{dt} \propto -\sqrt{h}$

$\frac{dV}{dt} = -k\sqrt{h}$ where k is a constant such that $k > 0$.

$V = \frac{8\pi h^{\frac{3}{2}}}{3}$

Many student did not use a constant of proportionality or did not indicate the sign of their constant.

$\frac{dV}{dh} = \frac{3}{2} \times \frac{8\pi h^{\frac{1}{2}}}{3}$

$\frac{dV}{dh} = 4\pi\sqrt{h}$

So we have

$$\frac{dV}{dt} = \frac{dV}{dh} \times \frac{dh}{dt}.$$

which gives

$$-k\sqrt{h} = 4\pi\sqrt{h} \times \frac{dh}{dt}$$

$$\frac{dh}{dt} = -\frac{k}{4\pi}$$

Which is a constant rate of fall.

(b)(i) Hearts Diamonds

$$^{13}C_2 \times {}^{13}C_3 = 22308.$$

(ii) Five cards of same suit.

Suit Ranks

$$^{4}C_1 \times {}^{13}C_5$$

Four cards of same suit

Suit Ranks 5th Card suit Rank

$$4C_1 \times {}^{13}C_4 \times {}^{3}C_1 \times {}^{13}C_1$$

At least four card of same suit.

$$({}^{4}C_1 \times {}^{13}C_5) + ({}^{4}C_1 \times {}^{13}C_4 \times {}^{3}C_1 \times {}^{13}C_1) = 116688$$

(c) (i) $\dot{y} = V\sin\theta - gt$

when $\dot{y} = 0$ $t = T$

$0 = V\sin\theta - gT$

$$T = \frac{V\sin\theta}{g}$$

Some student differentiated the $\sin\theta$ to $\cos\theta$ but this is a constant not a function of time.

(ii) when $t = T$ $y = H$.

$$H = VT\sin\theta - \frac{1}{2}gT^2.$$

$$H = \frac{V^2\sin^2\theta}{g} - \frac{1}{2}g\left(\frac{V^2\sin^2\theta}{g^2}\right)$$

$$H = \frac{V^2\sin^2\theta}{2g}$$

Find y when $t = T/2$

$$y = \frac{VT\sin\theta}{2} - \frac{1}{8}gT^2.$$

$$= \frac{V^2\sin^2\theta}{2g} - \frac{g}{8}\left(\frac{V^2\sin^2\theta}{g^2}\right)$$

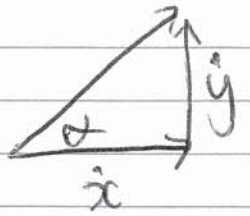
$$= \frac{V^2\sin^2\theta}{2g} - \frac{V^2\sin^2\theta}{8g}$$

$$y = \frac{3}{8}\,\frac{V^2\sin^2\theta}{g}.$$

But $$2H = \frac{V^2\sin^2\theta}{g}$$

$$y = \frac{3}{8} \times 2H$$
$$= \frac{3}{4}H.$$

(iii)



$$\dot{x} = V\cos\theta$$
$$\dot{y} = V\sin\theta - gt$$
$$= V\sin\theta - \frac{gT}{2}$$
$$= V\sin\theta - \frac{V\sin\theta}{2}.$$
$$\dot{y} = \frac{1}{2}V\sin\theta.$$

$$\tan\alpha = \frac{\dot{y}}{\dot{x}} = \frac{\frac{1}{2}V\sin\theta}{V\cos\theta}$$
$$= \frac{1}{2}\tan\theta.$$